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3-D Numerical Hydrodynamics of Self-Gravitating Gas

—— *Collisions and Fragmentations* ——

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ABSTRACT

The collisions between self-gravitating gas (*e.g.* interstellar clouds or stars) are investigated using the three dimensional Smoothed Particle Hydrodynamics. We solve the Euler equation coupled with the Poisson equation numerically. The formalism to calculate the energy equation and to include the radiation reaction of gravitational waves are also presented. In the gas system with Newtonian gravity, the criterion for gravitational instability is known as the Jeans criterion by linear perturbation theory. We simulate the nonlinear evolution and find the dynamical criterion different from that. In supersonic head-on collisions between two stable isothermal clouds, the shock compression increases the density and the self-gravity can trigger the instability or induce the quadrupole oscillation as expected in the tensor virial analysis. When we include the gas cooling effect, the cloud fragments into small pieces. In the case of off-center collisions, the outcomes depend on the nondimensional constant $q \equiv JC_s/GM^2$, where M is the total mass, J is the total angular momentum and C_s is the sound velocity of isothermal gas. If the parameter is small, $q \lesssim 0.2$, the shock compression triggers the gravitational collapse and the rapidly rotating core forms near the collisional center. The system with $q \gtrsim 0.4$ starts fission to form the binary cloud system after the collisional merging, For the intermediate case , they make a merged disk with a bar-spiral structure.

§1. Introduction

The self-gravity plays an important role in the universe and involves some difficult problems such as the many-body problem or the fission and fragmentation theory. The self-gravitating gas may be regarded as a kind of plasma without the Debye shielding since the gravity is always attractive. Although the two points motions are perfectly understood in Newtonian mechanics, the dynamical behavior of the gaseous system cannot be treated analytically. We want to know what kind of physical quantity decides the nonlinear evolution in this pure dynamical system. As an example of the interaction between two gravitationally bound states, we simulate the collision of interstellar clouds.

To understand the star formation from the interstellar clouds, we have to know the condition of gravitational instability. This corresponds to the onset of phase change from the diffuse state towards the condensed phase in which the density grows up to 10^{20} times higher. The fact that the uniform gaseous medium is gravitationally unstable against the long wavelength perturbations is known as the Jeans instability. While, the stability condition for the single hydrostatic equilibrium solution is the Bonnor-Ebert criterion,which indicates the maximum mass of the stable solution $M_{BE} = 1.18(C_s^8/G^3P_e)^{1/2}$. When the self-gravitating isothermal gas are compressed and the density increases, the maximum mass which is gravitationally stable is believed to be reduced. We investigate whether this idea is true or not and search the new criterion of stability in the dynamical processes.

In addition, many astrophysicists want to study on the origin of rotating astronomical objects. The angular momentum distribution is a free function as the initial condition for the evolution of rotating gas. This freedom is known to decide the structure of the axisymmetric equilibrium solutions. [1] We try to get the information about the initial distribution of angular momentum for the objects produced by the off-center collisions.

§2. Basic Equations

Hydrodynamics of the self-gravitating gas can be described by the Euler equations coupled with the Poisson equation for the gravitational potential . The mass conservation is written as the equation of continuity,

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \boldsymbol{v}) = 0 \quad . \tag{2.1}$$

In numerical calculations, the Euler equation should be solved with some artificial viscosity,

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla P - \nabla(\psi + \Psi) - \epsilon \boldsymbol{Q} \quad . \tag{2.2}$$

The Poisson equation determines Newtonian self-gravitational potential ψ,

$$\Delta\psi = 4\pi G\rho \quad . \tag{2.3}$$

The reaction from radiating gravitational waves causes the correction Ψ to Newtonian potential, [2)]

$$\Psi = \frac{G}{5c^5} \overset{(5)}{\boldsymbol{D}}_{\alpha\beta}\, x^\alpha x^\beta \quad , \tag{2.4}$$

using the quadrupole mass moment,

$$\boldsymbol{D}_{\alpha\beta} = \int \rho(x_\alpha x_\beta - \tfrac{1}{3}\delta_{\alpha\beta} x^\mu x_\mu) dV \quad . \tag{2.5}$$

For the ideal gas with the specific-heat ratio γ, the equation of state is

$$P = (\gamma - 1)\rho U \quad . \tag{2.6}$$

We have to solve the energy equation to decide the internal energy U of the gas. We have succeeded in simulating the adiabatic evolution, but many information from the atomic physics are still required to solve the radiative transfer equation or to include the cooling function of interstellar gas. In this paper, we report mainly on the nature of isothermal gas *i.e.* $P = \rho C_s^2$, because it makes the problem simple and is considered as a good approximation of the actual interstellar molecular clouds.

Different from the common fluid, the self-gravitating gas does not need the fixed boundary condition. (Can you imagine a box which holds the Sun ?) It makes the zero-density surface boundary by itself. For the soft gas whose polytropic index $n = 1/(\gamma - 1) > 5$, however, the boundary extends to infinity so that we assume the gas is surrounded by the external hot medium which exert the constant pressure P_e on the surface and neglect the gravity from such a tenuous medium. It is the case in the actual universe, for example, the molecular gas clouds are often surrounded by the ionized hot regions.

§3. Smoothed Particle Hydrodynamics

We treat the three dimensional initial value problem using the numerical code called Smoothed Particle Hydrodynamics.(3 , 4[3),4)] This scheme is a kind of Monte Calro method and the fluid system is treated as the ensemble of N-fluid elements and the motion of each element is described in Lagragian coordinates. This method has an advantage to treat the three-dimensional space easily compared with the Finite Difference Method. Each element is assumed to have the same mass m_0 and its own internal density distribution, for which we chose the Gaussian type smoothing kernel. The local density of fluid is given by the superposition of density distribution of all the elements,

$$\rho(\boldsymbol{x}_i) = \frac{m_0}{\pi\sqrt{\pi}} \sum_{j=1}^{N} \frac{1}{h_j^3} \exp(-|\boldsymbol{x}_i - \boldsymbol{x}_j|^2/h_j^2) \quad . \tag{3.1}$$

The smoothing length of i-th fluid element is determined locally in accordance with the spatial variation of density as

$$h_i = \eta \left(\frac{m_0}{\rho(\boldsymbol{x}_i)} \right)^{1/3} \quad , \tag{3.2}$$

where η is the coefficient which determines the resolution. The gas motion is described by the equation of motion for i-th fluid element

$$\frac{d\boldsymbol{v}_i}{dt} = -\frac{1}{\rho(\boldsymbol{x}_i)} \nabla P(\boldsymbol{x}_i) - \nabla\psi(\boldsymbol{x}_i) - \epsilon \sum_{j=1}^{N} \boldsymbol{Q}_{ij} \quad . \tag{3.3}$$

We notice that the basic Partial Differential Equations are converted to Ordinary

Differential Equations.

The components of Newtonian gravity can be calculated directly by integrating the mass distribution instead of solving the Poisson equation.

$$\nabla\psi(\boldsymbol{x}_i) = Gm_0 \sum_{j=1}^{N} \frac{(\boldsymbol{x}_i - \boldsymbol{x}_j)}{x_{ij}^3} \left[\mathrm{erf}(\frac{x_{ij}}{h_j}) - \frac{2}{\sqrt{\pi}} \frac{x_{ij}}{h_j} \exp(-x_{ij}^2/h_j^2) \right] \quad , \tag{3.4}$$

where $x_{ij} \equiv |\boldsymbol{x}_i - \boldsymbol{x}_j|$.

§4. Energy Equations

To our SPH, we apply the energy equation developed for Particle-and-Force (PAF) method, [5] and have tested the code in the case of an adiabatic shock tube. Kinetic energy per unit mass of i-th element is

$$K_i = \tfrac{1}{2}|\boldsymbol{v}_i|^2 \quad . \tag{4.1}$$

The energy change rate of each particle should be given by the work that the other particles do on it. The power is given by the product of force using the mean value of each pair velocities as

$$\frac{d}{dt}(K_i + U_i) = \sum_{j\neq i}^{N} \boldsymbol{F}_{ij} \cdot \left(\frac{\boldsymbol{v}_i + \boldsymbol{v}_j}{2} \right) \quad , \tag{4.2}$$

where $\boldsymbol{F}_{ij}$ is the pressure gradient and viscosity force exerted by i-th particle onto j-th particle. This definition satisfies the energy conservation for the system in which there is no external force. If all the interparticle force functions satisfy the momentum conservation ($\boldsymbol{F}_{ij} = -\boldsymbol{F}_{ji}$), then the total energy conservation is guaranteed as

$$\frac{dE}{dt} = \frac{d}{dt}\sum_{i=1}^{N}(K_i + U_i) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}^{N} \boldsymbol{F}_{ij} \cdot (\boldsymbol{v}_i + \boldsymbol{v}_j) = 0 \quad . \tag{4.3}$$

From the equation of motion, we know that

$$\frac{dK_i}{dt} = \boldsymbol{v}_i \cdot \frac{d\boldsymbol{v}_i}{dt} = \boldsymbol{v}_i \cdot \sum_{j \neq i}^{N} \boldsymbol{F}_{ij} \quad . \tag{4.4}$$

Therefore, we get the energy equation which calculates the change of fluid temperature,

$$\frac{dU_i}{dt} = \frac{1}{2} \sum_{j \neq i}^{N} \boldsymbol{F}_{ij} \cdot (\boldsymbol{v}_j - \boldsymbol{v}_i) \quad . \tag{4.5}$$

This equation can be rewritten in the form eliminating the negative internal energy as,

$$\frac{dU_i}{dt} = U_i \sum_{j \neq i}^{N} \frac{1}{U_i + U_j} \boldsymbol{F}_{ij} \cdot (\boldsymbol{v}_j - \boldsymbol{v}_i) \quad . \tag{4.6}$$

§5. Numerical Results

As the initial condition, we assume the hydrostatic clouds of mass M collide with the relative velocity V and the impact parameter b. In the collision with the relative velocity greater than the sound velocity, the shock-compressed layer can be formed. In this supersonic interactions, the main difficulty is concerning to the choice of artificial viscosity. The pure particle scheme cannot avoid the particle penetration. [6] Another SPH using the constant smoothing length h is in a limited success to reproduce the analytic shock condition in 3-D collisions. [7] With the same viscosity used in the paper of Miyama *et al.*, [8] we reproduce the central density increase such as $\rho/\rho_0 \sim (V/C_s)^2$ for the head-on collisions in the range of $V \lesssim 6C_s$ using N=8000 particles (Fig.1).

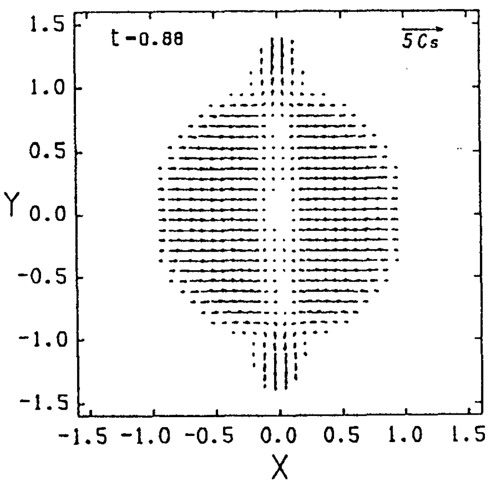

Fig.1. The velocity vectors in the x-y plane at the shock formation stage. Clouds collide along the x-axis with $V = 5Cs$.

As a result, the gravitational instability is induced in our simulation and the central part of the cloud begins to collapse even if the total mass is as small as $\sim 0.8 M_{BE}$. The collapsing model shows the similarity density profile $\rho \propto r^{-2}$, which is typical to the isothermal collapse (Fig.2). From the forty calculated models, [9] we find that disruption or fragmentation by the isothermal collision is less likely and the sticking probability is very high. The typical evolution of the stable head-on collision is the oblate-prolate oscillation which leads to the new hydrostatic equilibrium state. The period of oscillation is in agreement with the eigen frequency of the quadrupole oscillation of the compressible self-gravitating gas. [9]

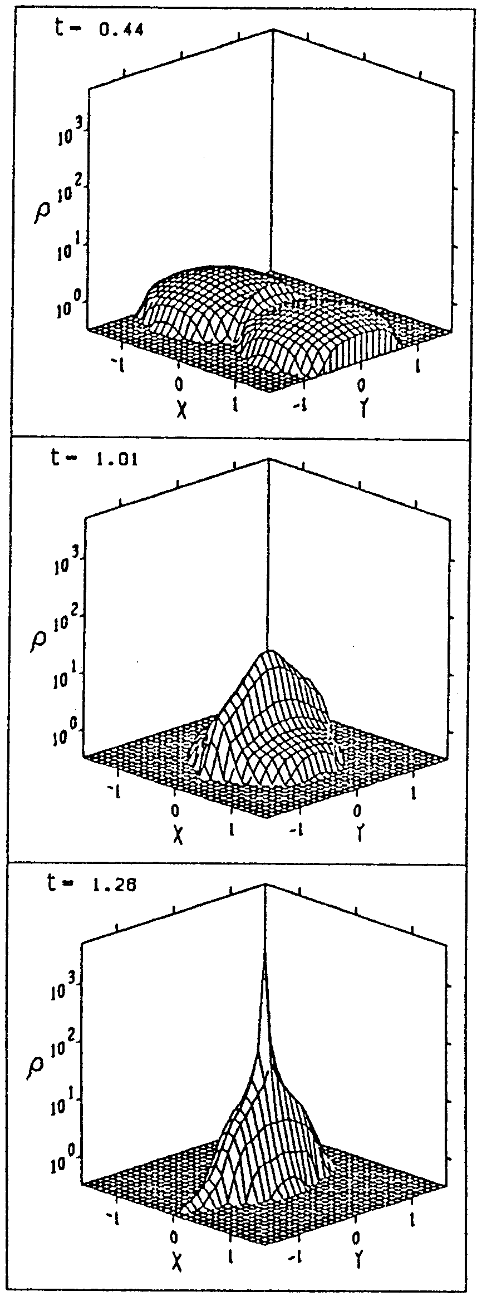

Fig.2. The evolution of density in the x-y plane for the triggered collapse model ($M = 1.13\ M_{BE}$, $V = 3C_s$, $b = 0$).

In the case of off-center collisions, the shock structure does not affect the nonlinear evolution. The outcomes depend on the nondimensional parameter written with constants of motion, $q \equiv JC_s/GM^2$. The linear analysis [10] and three dimensional simulations of dynamics of rotating isothermal clouds [8] also indicate the importance of this parameter. If $q \lesssim 0.2$ the angular momentum is not sufficient to stop the gravitational collapse. The contraction proceeds forming the rapidly rotating core near the collisional center. With slightly large angular momentum, $0.2 \lesssim q \lesssim 0.4$, the collision makes a merged disk with a bar-spiral structure as in Fig.3. The system with $q \gtrsim 0.4$ starts fission to appear as the binary cloud system after the collisional merging.

For the case of strong oblique shock, the shocked region reduce the collisional velocity and becomes a rigidly rotating core, while the unshocked region extends as the halo in Kepler rotation. In the case of collapse triggered by the weak shock, the density shows the similarity profile typical to the isothermal collapse and the flat rotation curve $v_\phi \approx const$ appears in the outer envelop. In this way, the merged system gets the spin angular momentum which results from the initial orbital angular momentum. The total angular momentum is always conserved, but the distribution of the specific angular momentum changes under influence of the non-axisymmetric process, that is, the gravitational torque. In the merged cloud made by the off-center collision, there exists the strong non-axisymmetric bar mode perturbation and it continues to transfer the specific angular momentum. That means the central part gets the higher density and the outer envelop extends the arm-spiral structure easily.

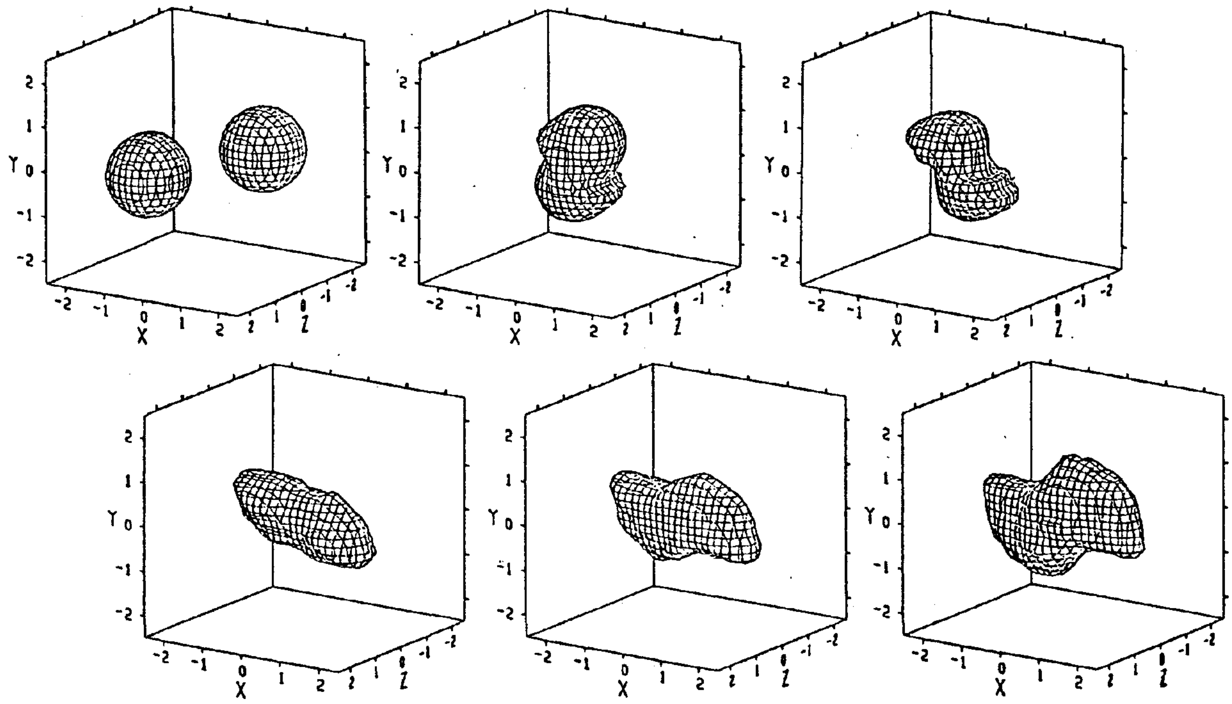

Fig.3. The equidensity surface of the stable merging model after the off-center collision ($q \sim 0.2$). The first disk is made by the shock compression, then it changed the flattening direction due to the angular momentum and makes a rotating disk with a bar structure.

In addition, we simulate the tidal encounter without direct collision and found that tidal torques make the initially spinless clouds start rotation even if there is no viscosity. The induced spin angular momentum is about several percents of the total angular momentum of the system.

To investigate the effect of equation of state, we study the colliding model whose polytropic index is assumed as $\gamma \sim 0.73$ according to the radio observational data of molecular clouds. [11] Such a kind of gas cools down by the compression so that the very thin disk forms at the collisional interface. Due to the gravitational instability of thin disk, the fragmentation proceeds and many small clumps with the size comparable to the disk thickness appear as shown in Fig.4. The central part continues collapsing, while the other clumps remain as a group of small cloudlets.

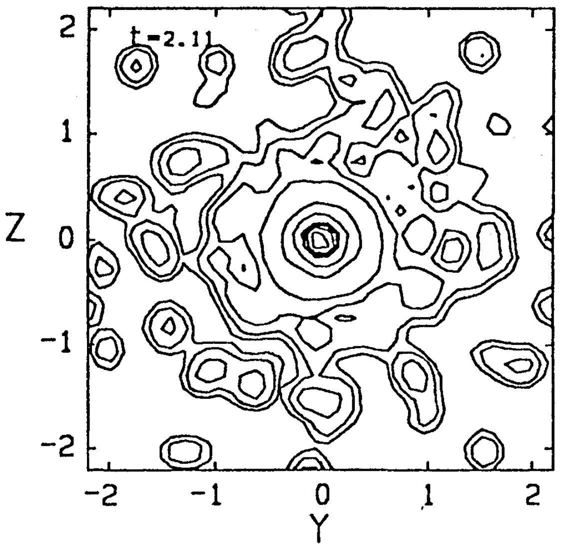

Fig.4. The density contours in the y-z plane for the cooling model. The temperature decrease down to about a third of initial value and creates many clumps.

As in the experiments of elementary particle physics, there are many other physical parameters in the simulation of collisions between the self-gravitating gas : mass ratio, initial spin motion and its direction relative to orbital angular momentum and so on. We will go on this survey developing the code which can solve the adiabatic collision to simulate the evolution of stellar system. 3-D post-Newtonian hydrodynamic code is also planned to calculate the gravitational waves and its back-reactions to the fluid motion. We think that the numerical solutions can help us to understand the nature of nonlinear interaction in the self-gravitating system. However, the analytic solutions are more valuable even with some strong assumptions. In order to confirm the validity of the numerical solutions, the analytic nonlinear solutions in dynamical processes are required in our field of astrophysics.

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